

ORIGINAL ARTICLE

# Recognition Thresholds for Letters with Simulated Dioptric Blur

HIROMI AKUTSU, PhD, HAROLD E. BEDELL, PhD, and SAUMIL S. PATEL, PhD

*University of Iowa, Department of Neurology, Iowa City, Iowa (HA) and University of Houston, College of Optometry, Houston, Texas (HEB, SSP)*

**ABSTRACT:** We examined the effects of simulated dioptric blur on the degradation of visual acuity using digitally filtered letters. Four types of digital filters were applied to 5 letters (C, D, E, O, and S), constructed to the specifications of Sloan optotypes. These filters were: (1) "normal," designed to simulate the positive and negative lobes of the modulation transfer function (MTF) produced by dioptric blur; (2) "truncated," which passed only those spatial frequencies up to the first zero of the MTF; (3) "phase-rectified," which inverted all of the negative lobes of the MTF to positive; and (4) "truncated-plus-negative," which eliminated all positive lobes above the first zero of the MTF. The letter size required to achieve 60%-correct identification was determined for letters that were filtered to simulate +1, +2, and +4 D of blur. Letters subjected to normal, truncated, and truncated-plus-negative filtering had approximately the same acuity threshold, whereas the threshold size for phase-rectified letters was significantly better. Our interpretation of these results is that dioptric blur hinders letter recognition because useful spatial frequency information is limited to that below the first zero of the MTF, and not because of interference from the phase-reversed spatial frequency information above the first zero. Our letter identification thresholds are consistent with recent evidence that the critical information for letter acuity corresponds to approximately 1.5 cycles/letter. (*Optom Vis Sci* 2000;77:524-530)

**Key Words:** visual acuity, defocus, spurious resolution, modulation transfer function

Dioptric blur that results from improper lens correction or inaccurate accommodation, particularly in older adults, is a common cause of poor spatial vision. The optical modulation transfer function (MTF) for dioptrically blurred targets is characterized by a series of negative and positive lobes above the first zero crossing,<sup>1-4</sup> the spatial frequency at which the MTF first crosses the spatial frequency axis (first zero; see Fig. 1A). The negative and positive lobes of the MTF above the first zero crossing comprise the region of spurious resolution, the range of which depends on the amount of dioptric blur, the pupil size, the wavelength composition of the stimulus, and the type and extent of coexisting ocular aberrations.<sup>1, 4-6</sup> Because the spatial frequencies contained in the negative lobes of the MTF undergo a phase reversal, dioptric blur could distort the shape of letter targets and render them unreadable (cf., Thorn & Schwartz<sup>7</sup>). Alternately, letter acuity may worsen with increasing amounts of dioptric blur simply because of the resulting shift of the resolution limit to lower spatial frequencies.

Dioptric blur has been shown to degrade visual acuity for letter targets substantially more than for sinusoidal gratings.<sup>7-9</sup> For example, Thorn and Schwartz<sup>7</sup> reported letter acuities of approxi-

mately 20/50, 20/130, and 20/300 with +1, +2, and +4 D of lens-induced blur. In comparison, grating acuities were approximately 20/30, 20/40, and 20/50 for the same amounts of lens blur. These substantial differences in acuity for letter vs. grating targets have been accounted for by stipulating that the identification of blurred letters is limited by the spatial frequency components below the first zero of the MTF, whereas the detection of blurred gratings can be achieved using any suprathreshold component of the MTF (e.g., refs.<sup>5, 7, 8, 10</sup>). However, it remains unclear whether the reversal of spatial phase information for frequencies above the first zero crossing of the MTF contributes directly to the degradation of visual acuity for blurred letters.

Because the MTF is a linear filter that is applied before the formation of the retinal image, manipulations of the spatial frequency spectrum of the visual target can mimic any arbitrary MTF.<sup>11, 12</sup> By comparing size thresholds for letter recognition using different types of digitally filtered letters, we attempted to evaluate the influence of spurious resolution on letter recognition. Four types of digital filters were applied: (1) a "normal" filter, which simulated the positive and negative lobes of the MTF in the presence of dioptric blur; (2) a "truncated" filter, which passed

only spatial frequencies up to the first zero crossing of the MTF; (3) a “phase-rectified” filter, which inverted all negative lobes of the MTF to positive; and (4) a “truncated-plus-negative” filter, which eliminated all positive lobes above the first zero crossing of the MTF. The MTFs for these four filters are shown for +2 D of simulated dioptric blur and a 3-mm pupil in Fig. 1.

The rationale of our experiments is the following: if spurious resolution impairs letter recognition, then letter acuity should improve when spatial frequencies above the first zero crossing are removed from the MTF with the truncated filter. On the other hand, if the recognition of blurred letters depends only on the spatial frequency of the first zero crossing, then application of the truncated filter should yield the same threshold letter size as with the normal filter. When the phase-rectified filter is used, letter acuity may improve because (1) inversion of the negative lobes of the MTF in the region of spurious resolution effectively increases the resolution limit and/or (2) the negative lobes of the normal MTF are eliminated. Finally, the degradation of letter recognition by the negative lobes of the normal MTF could be ameliorated by the nonphase-reversed spatial frequency information in the positive lobes above the first zero crossing of the MTF. If so, then application of the truncated-plus-negative filter, which eliminates all positive lobes above the first zero crossing, should yield poorer letter acuity than either the normal or truncated filters.

## METHODS

### Stimuli

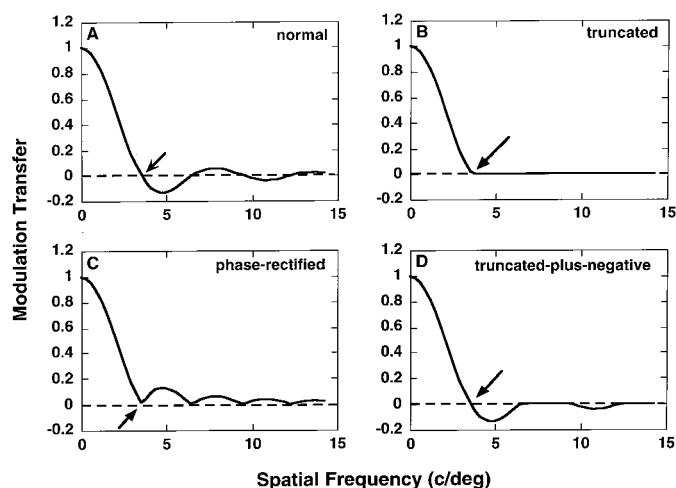
Five letters (C, D, E, O, and S) were constructed in a  $5 \times 5$  matrix according to Sloan's<sup>13</sup> specifications with letter strokes one unit in width. Dark letters were presented on a bright background ( $35 \text{ cd/m}^2$ ) with a color computer monitor (Apple Multiscan, 15 inches) at a frame rate of 75 Hz. The wavelength composition of

the screen was restricted by using only the green gun of the color monitor. Ten sizes of each letter were prepared.

Observers were tested with unfiltered letters presented with 0, +1, +2, and +4 D spectacle lens blur and with four types of digitally filtered letter stimuli. First, MTFs for an aberration-free human eye with +1, +2, and +4 D of dioptric blur were implemented digitally in MATLAB software (The MathWorks, Natick, MA), assuming a 3.0 mm pupil and midspectral light (see Appendix 1). Three permutations of these MTFs were then produced by: (1) setting all values above the first zero crossing to zero (truncated MTF); (2) setting the sign of all MTF values above the first zero crossing to positive (phase-rectified MTF); or (3) setting all positive values above the first zero crossing to zero (truncated-plus-negative MTF). Normal, truncated, phase-rectified, and truncated-plus-negative letters with +1, +2, and +4 D of simulated blur were constructed by multiplying the appropriate MTF by the Fourier transform of each unfiltered Sloan letter, followed by an inverse Fourier operation. Filtered versions of the 10 different letter sizes were constructed by taking into account the viewing distance for each level of simulated blur (see Table 1). The digital MTFs had a resolution along the horizontal and vertical spatial frequency axes of 1 (+1 D blur) or 0.5 (+2 and +4 D blur) cpd. Based on the viewing distance and pixel size (0.03 cm), the maximum spatial frequency represented in each filtered letter corresponded to 247 (+1 D) or 123 (+2 and +4 D) cpd. Digital images had a contrast resolution of 8 bits, and the screen was carefully gamma-corrected using a technique described by Pelli and Zhang.<sup>14</sup> Luminances were measured using a Minolta LS 110 luminance meter and stimulus contrast is specified in terms of the Weber definition ( $\Delta L/L_{\text{background}}$ ).

### Procedures

Each observer viewed the letter stimuli monocularly through a 3.0-mm artificial pupil, placed in a trial frame, using the preferred eye. For experiments with lens-defocused letters, blur was produced by having the observer view the computer monitor through the artificial pupil and an appropriate trial lens. Refractive correction was also implemented by adding appro-



**FIGURE 1.**

MTFs of the four digital filters used in this study with +2 D of simulated dioptric blur and a pupil diameter of 3 mm. A: normal filter, B: truncated filter, C: phase-rectified filter, D: truncated-plus-negative filter. Although the MTFs shown in the figure are continuous, the digital MTFs for +2 D of blur were sampled with a resolution of 0.5 cpd. The arrow in each panel indicates the location of the first zero of the MTF.

**TABLE 1.**

Angular letter sizes for each blur condition.

| Filter | +1 D             | +2 D, +4 D       |
|--------|------------------|------------------|
|        | Viewing Distance | Viewing Distance |
| Letter | Angular Size     |                  |
|        | (min arc)        | (min arc)        |
| 1      | 7.8              | 9.7              |
| 2      | 9.7              | 12.1             |
| 3      | 12.1             | 15.5             |
| 4      | 15.5             | 19.4             |
| 5      | 19.4             | 24.3             |
| 6      | 24.3             | 31.1             |
| 7      | 31.1             | 38.8             |
| 8      | 38.8             | 48.5             |
| 9      | 48.5             | 62.1             |
| 10     | 62.1             | 78.1             |

appropriate trial lenses, if necessary. The viewing distance varied between 4.25 and 8.5 m to achieve an appropriate range of letter sizes for each level of letter defocus. For experiments with filtered letters, the observer viewed the screen through the same 3.0-mm artificial pupil with no trial lenses, except those required to correct refractive error.

Letters of five different sizes were presented in random order using the method of constant stimuli. The unfiltered letters had a contrast of 75%. Peak contrast of the filtered letters was lower because of the contrast reduction that accompanied filtering. Within each block of trials, letter size was changed from trial to trial while the viewing distance remained fixed. Letters were presented for 500 ms, with abrupt onset and offset. After each presentation, the observer made a forced choice response using one of five adjacent keys on the computer keyboard, each labeled clearly with one of the tested letters. A psychometric function was constructed from each block of 100 trials (5 letters  $\times$  5 sizes  $\times$  4 presentations/letter) and subsequently fit by probit analysis.<sup>15</sup> The letter size corresponding to the recognition threshold was the 60% point of the psychometric function, halfway between chance (20%) and perfect (100%) performance.

A preliminary experiment, to compare the contrast thresholds for detecting blurred and normal filtered letters, was conducted using two observers (KL and YS). Contrast detection thresholds were measured for the letters “O” and “E,” with sizes equal to 15.5 min arc for +1 D, 24.3 min arc for +2 D, and 48.5 min arc for 4 D of lens-induced or simulated blur. These letter sizes corresponded to the acuity limit for each level of defocus in a pilot experiment. A temporal 2AFC method in conjunction with the QUEST algorithm<sup>16</sup> estimated the level of contrast that yielded 82% correct responses. Thresholds for +1, +2, and +4 D of optical and simulated blur were determined for each observer in random order.

Letter-size thresholds for unfiltered letters viewed with various amounts of lens-induced blur were measured first in each observer. The order of the first three filtered-letter (normal, truncated, and phase-rectified) conditions was randomized among observers, and approximately counterbalanced within each observer. Thresholds for truncated-plus-negative filtered letters were determined for two observers (KL and TN) after all of the other acuity conditions were completed. Between two and four measures of threshold were taken in separate sessions for each test condition and the averaged measures are reported. Representative psychometric functions for observer TN are shown for each of the +2 D filter conditions in the insets to Figs. 3 and 4.

## Observers

The five observers (EK, KL, QP, TN, and YS) were experienced in visual psychophysical experiments, but naive as to the purpose of this study. The ages of the observers ranged from 22 to 30 years old. All had corrected-to-normal visual acuities in both the tested and untested eye. The refractive corrections for each observer's tested eye are shown in Table 2. None had any astigmatic refractive error. Voluntary informed consent was obtained from each observer before data collection began.

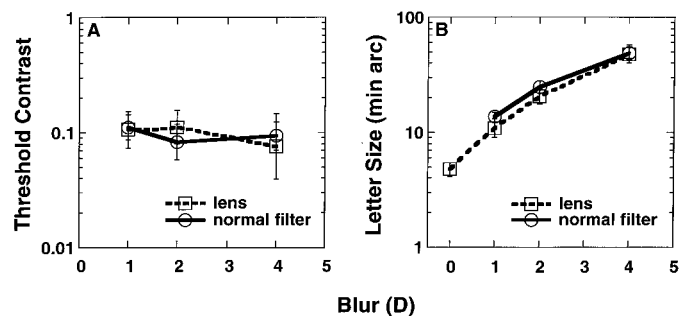


FIGURE 2.

A: Contrast detection thresholds are plotted for spatially filtered letters (normal filter) and for lens-induced blur of +1 to +4 D. Data points are the average of two observers for two different letters (E and O). Error bars denote  $\pm 1$  SD. Letter size was set approximately to the size threshold for letter identification at each level of defocus. B: Size thresholds for letter identification (min arc) are plotted for spatially filtered letters (normal filter) and for lens-induced blur of 1 to 4 D. Note that the y axis gives the size of the whole letter at the acuity threshold. Data points are the average of five observers; error bars denote  $\pm 1$  SD.

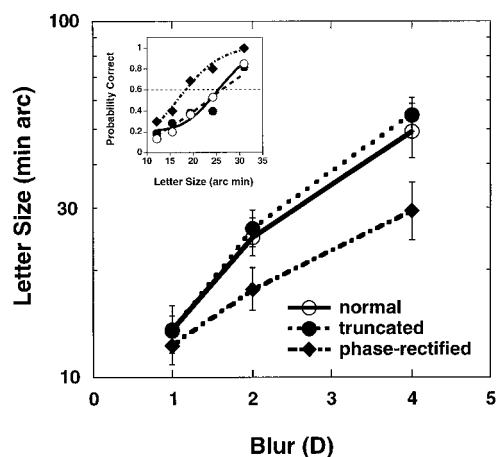


FIGURE 3.

Size thresholds (whole letter in min arc) for letter identification are plotted as a function of simulated blur in diopters for three different types of spatial filter. Each data point represents the average for five observers; error bars denote  $\pm 1$  SD. The inset shows the psychometric functions (probability correct vs. letter size) of observer TN for each type of filter with +2 D of simulated defocus. The symbols and line styles in the inset correspond to the same filter conditions depicted in the main figure.

TABLE 2.

Refractive correction for the tested eye of each observer.

| Observer   | EK        | KL        | QP        | TN        | YS        |
|------------|-----------|-----------|-----------|-----------|-----------|
| Refraction | -1.25 sph | -0.75 sph | -0.75 sph | -0.50 sph | -3.00 sph |

## RESULTS

### Comparisons between filtered letters and optically blurred letters

We made two comparisons to verify that observers responded similarly to our digitally filtered letters and to letters with lens-induced blur. First, contrast detection thresholds were compared for normal filtered letters with +1, +2, and +4 D of simulated blur and for letters blurred optically by the same amounts. Second,

size thresholds to identify +1, +2, and +4 D normal filtered letters were compared to the size thresholds for identifying letters that were blurred optically.

Contrast detection thresholds, averaged for the two observers across both letters (4 replications per condition), are plotted in Fig. 2A. A three-way repeated measures analysis of variance (ANOVA) (for defocus type, magnitude of defocus, and letter) indicated that log contrast varied slightly but significantly with the magnitude of defocus ( $F_{2,2} = 22.99$ ;  $p = 0.042$ ). A significant difference in contrast thresholds was also found between the letters E and O (letter E < letter O) ( $F_{1,1} = 1738.1$ ;  $p < 0.015$ ). However, there was no significant effect of the type of defocus ( $F_{1,1} = 0.006$ ;  $p > 0.94$ ), no significant interaction between the type and amount of defocus ( $F_{2,2} = 2.16$ ;  $p > 0.30$ ), and no significant interaction between the type of defocus and the letter ( $F_{1,1} = 1.00$ ;  $p > 0.49$ ). These results indicate that similar amounts of contrast are required to detect letters that have comparable amounts of digitally simulated and lens-induced dioptric blur.

The size thresholds for letter recognition, averaged across five observers, are slightly poorer for letters with simulated than with lens-induced blur, using the +1, +2, and +4 D normal filters (Fig. 2B). In spite of this small but significant difference between simulated and lens-induced blur ( $F_{1,4} = 14.53$ ;  $p < 0.02$ ), it should be noted that there are no interactions between the type and amount of blur ( $F_{2,8} = 3.66$ ;  $p > 0.07$ ). Therefore, the results obtained in our main experiments with simulated blur can be generalized to lens-induced blur simply by adding a small offset on the blur axis. The difference in recognition thresholds for simulated vs. lens-induced blur might result from approximations in our construction of the digital MTFs or from interactions between lens blur and the ocular aberrations of our observers.<sup>17</sup>

### Comparisons between digitally filtered letters

Size thresholds for letter identification are virtually identical for letters filtered with the normal and truncated MTFs, as seen in Fig. 3. However, the size thresholds for the phase-rectified letters were lower than for the other two types of filtered letters. These results were consistent across all five observers.

To evaluate differences among these three types (normal, truncated, and phase-rectified) of digitally filtered letters, we performed a two-way, repeated-measures ANOVA on the log-transformed letter sizes at threshold. The effect of filter type was significant ( $F_{2,8} = 76.79$ ;  $p < 0.001$ ), as was the interaction between the filter type and the amount of defocus ( $F_{4,16} = 22.17$ ;  $p < 0.001$ ). *Post hoc* tests using Tukey's HSD found significant differences ( $p < 0.05$ ) between the threshold letter sizes for the phase-rectified letters and the other filtered letters at +2 and +4 D defocus. There was no significant difference between the size thresholds for the normal and truncated filtered letters for any magnitude of defocus.

Size thresholds for letter identification were also virtually identical for letters filtered with the normal and truncated-plus-negative MTFs, as seen in Fig. 4. A repeated-measures ANOVA performed on the log-transformed letter sizes at threshold found no significant difference between the recognition thresholds using these two MTFs ( $F_{1,1} = 7.25$ ;  $p > 0.2$ ).

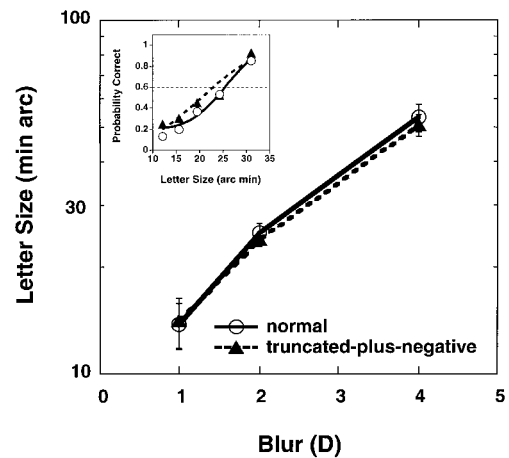


FIGURE 4.

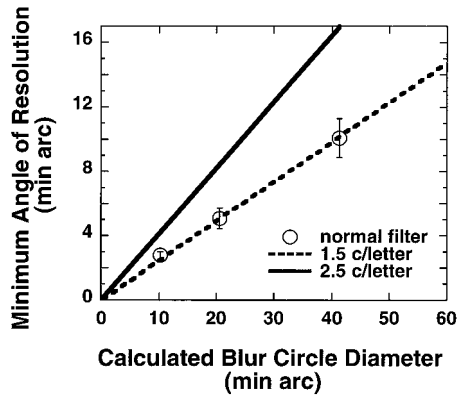
Size thresholds (whole letter in min arc) for letter identification are plotted as a function of simulated blur in diopters, using normal and truncated-plus-negative filters. The data points are the average for two observers; error bars denote  $\pm 1$  SD. The inset shows the psychometric functions (probability correct vs. letter size) of observer TN for +2 D of simulated defocus. The symbols and line styles in the inset correspond to the same filter conditions shown in the main figure.

### DISCUSSION

A theoretical approximation that relates pupil diameter, amount of defocus, and the diameter of the blur circle on the retina was derived by Smith<sup>5</sup> and is described by equation A1 in Appendix 1. Subsequently, Smith et al.<sup>12</sup> determined that the minimum angle of resolution (MAR, in min arc) for Landolt rings varies approximately linearly with the size of the retinal blur circle (also expressed in min arc) with a slope of 0.26. Our acuity results for the normal filter condition exhibit approximately the same change in MAR with the size of the retinal blur circle that is calculated using equation A1. Specifically, the slope of the line fitted to our data is 0.24, compared to the slope of 0.26 obtained by Smith et al.<sup>12</sup> (see Fig. 5).

Theoretically, the relationship between letter MAR and the size of the retinal blur circle is expected to have a slope of 0.41 (shown by the solid line in Fig. 5; also see Appendix 2) when both are expressed in min arc. This theoretical relationship is based on two principal assumptions: (1) that letter acuity is limited by the resolution of spatial frequency components corresponding to 2.5 cycles/letter (e.g., that letter acuity of 20/20 and resolution of a 30 cpd grating are both equivalent to a MAR of 1 min arc) and (2) that the resolution limit is set by the first zero crossing of the MTF.

This sizable discrepancy between the slopes of the empirical (approximately 0.25) and theoretical (0.41) relationships could be accounted for (1) if letter acuity depends on the resolution of spatial frequency components less than 2.5 cycles/letter and/or (2) if letter acuity is impaired by aberrant information from spatial frequencies above the first zero crossing of the MTF. The results of our experiment show that visual acuity is unaffected by the presence or absence of spatial frequency components in the normal MTF above the first zero crossing. Consequently, we conclude that visual acuity for letters that are blurred up to at least 4 D depends only on the spatial frequency of the first zero crossing of the corresponding MTF.



**FIGURE 5.**

Size thresholds for the identification of normal filtered letters are converted to minimum angles of resolution (in min arc) and plotted against the calculated size of the retinal blur circle for +1, +2, and +4 D of simulated blur (in min arc). Error bars denote +1 SD. The best fit line to the data points has a slope of 0.24 (not shown in the figure), which overlaps the line for 1.5 cycles/letter. The two lines are predictions from equation A10 in Appendix 2, assuming that letter identification requires the resolution of a spatial frequency corresponding to either 1.5 (broken line) or 2.5 (solid line) cycles/letter.

In light of this conclusion, our acuity results suggest that the critical spatial frequency required to recognize optically blurred letters corresponds to approximately 1.5 cycles/letter (from equation A10, the critical spatial frequency =  $0.24 \times 5 \times 3.83/\pi = 1.46$ ). This inference is consistent with the outcome of several previous studies.<sup>18–23</sup> Thorn and Schwartz<sup>7</sup> concluded that the critical spatial frequency for letter identification is approximately 1.25 to 1.75 cycles/letter for relatively small amounts of blur, such as the range that was simulated in our study. With larger amounts of blur (e.g.,  $\geq +6$  D), the critical spatial frequency approached 2.5 cycles/letter (also see Alexander et al.<sup>21</sup>), which Thorn and Schwartz<sup>7</sup> attributed to greater visibility of these high letter-frequency components in very large letters, based on the shape of the human contrast sensitivity function.

Visual acuity is better for letters filtered using the phase-rectified version of the MTF than for letters subjected to the normal or truncated filters (see Fig. 3). Consequently, spatial frequencies above the first zero crossing of the (pre-rectified) MTF apparently can provide useful information for letter identification. However, this result is not surprising; if a smooth curve were fit to the phase-rectified MTF, it would gradually approach zero modulation at a spatial frequency above the nominal resolution limit. Presumably, the recognition of phase-rectified letters depends on the modulation of spatial frequencies that are available in the filtered image, weighted by the observer's contrast sensitivity function. Our finding that the difference in acuity for phase-rectified vs. normal (or truncated) filtered letters increases with the amount of simulated blur (see Fig. 3) is in qualitative agreement with this presumption. With greater amounts of simulated blur, the spatial frequencies in the "rectified" MTF that are above the nominal resolution limit will be closer to the peak of a normal observer's contrast sensitivity function.

Recently, Anderson and Thibos<sup>24</sup> investigated whether alias spatial frequencies contribute to the degradation of peripheral letter acuity. The alias spatial frequencies are thought to be produced

by the neural undersampling of high spatial frequency information that is available in the peripheral retinal image. When spatial frequencies above the sampling-based resolution limit were removed by digital spatial filtering, they found no improvement in peripheral visual acuity. Analogous to our conclusion for optically blurred targets in the fovea, they concluded that peripheral visual acuity depends only on the (neural) resolution limit and that the peripheral letter acuity threshold corresponds to a spatial frequency less than 2.5 cycles/letter.

In the introductory section of our paper, we noted that the location of the first zero crossing of the MTF depends on the wavelength composition of the stimulus and the extent of ocular aberrations, in addition to the amount of dioptric blur and pupil size. In our study, we attempted to minimize the influence of the wavelength composition of the stimuli by restricting illumination to mid-spectral (albeit, clearly not monochromatic) light. We attempted to minimize the contribution of ocular aberrations by using a relatively small artificial pupil. Although the pattern of acuity results is likely to become more complex in the presence of substantial chromatic and/or achromatic aberrations (e.g., Bradley et al.<sup>6</sup>), we nevertheless expect that letter acuity will continue to depend on the first zero crossing of the MTF, with no significant deleterious influence from spatial frequencies in the region of spurious resolution.

## APPENDICES

### Appendix 1

*Calculation of normal MTFs.* The derivations shown in these appendices are based on the geometrical optical approximations presented by Charman and Jennings<sup>4</sup> and Smith.<sup>5</sup> The approximate relationship between blur circle diameter, pupil diameter, and defocus is given below:

$$\phi_{\text{rad}} = pD \quad (\text{A1})$$

where  $\phi_{\text{rad}}$  is blur circle diameter in radians,  $p$  is the pupil diameter in meters, and  $D$  is the defocus in diopters. The approximate MTF of a defocused system with a circular pupil is given below:

$$\text{MTF}(\omega_m) = \frac{2J_1(\pi\phi_m\omega_m)}{\pi\phi_m\omega_m} \quad (\text{A2})$$

where  $\omega_m$  is the spatial frequency in cycles/m and  $J_1(\cdot)$  is the first order Bessel function. If we use angular spatial frequency ( $\omega_{\text{cpd}}$ ) in cpd in the above equation we have:

$$\text{MTF}(\omega_{\text{cpd}}) = \frac{2J_1(\pi\phi_{\text{deg}}\omega_{\text{cpd}})}{\pi\phi_{\text{deg}}\omega_{\text{cpd}}} = \frac{2J_1(180\phi_{\text{rad}}\omega_{\text{cpd}})}{180\phi_{\text{rad}}\omega_{\text{cpd}}} \quad (\text{A3})$$

where  $\phi_{\text{deg}}$  is the diameter of the blur circle in degrees. Substituting equation A1 in equation A3 we obtain:

$$\text{MTF}(\omega_{\text{cpd}}) = \frac{2J_1(180pD\omega_{\text{cpd}})}{180pD\omega_{\text{cpd}}} \quad (\text{A4})$$

The normal filters used in our study (e.g., see Fig. 1A) were constructed using equation A4.

## Appendix 2

Theoretical relationship between MAR and the diameter of the retinal blur circle. An approximate transformation of blur circle diameter from linear to angular units ( $\phi_{\text{rad}}$ ) is given by the formula:

$$\phi_{\text{rad}} = \frac{\phi_m}{l_0} \quad (\text{A5})$$

where  $\phi_m$  is blur circle diameter and  $l_0$  is the distance of the nodal point from the retina, both in meters. MTF( $\omega_m$ ), which is defined in equation A2, is zero when the Bessel function  $J_1(\cdot)$  becomes zero. The first zero of  $J_1(\cdot)$  occurs when its argument is 3.83. Therefore, the spatial frequency ( $\omega_{m0}$ ) corresponding to the first zero of the MTF( $\omega_m$ ) (resolution limit) is given by:

$$\omega_{m0} = \frac{3.83}{\pi \phi_m} \quad (\text{A6})$$

The angular spatial frequency ( $\omega_{\text{rad}0}$ ) in cycles/radian corresponding to  $\omega_{m0}$  is given by:

$$\omega_{\text{rad}0} = \omega_{m0} l_0 \quad (\text{A7})$$

Using equations A5, A6, and A7, the equation for  $\omega_{\text{rad}0}$  can be rewritten as follows:

$$\omega_{\text{rad}0} = \frac{3.83}{\pi \phi_{\text{rad}}} = \frac{3.83 \times 180 \times 60}{\pi^2 \phi_{\text{min}}} \quad (\text{A8})$$

where  $\phi_{\text{min}}$  is the diameter of the blur circle in min arc. Note that equation A8 is only valid when  $\phi_{\text{min}}$  is nonzero.

The resolution limit ( $\omega_{\text{rad}l}$ ) in cycles/radian as it applies to letter recognition is given as:

$$\omega_{\text{rad}l} = \frac{180}{\pi} \left( \frac{60\text{CPL}}{5\text{MAR}} \right) \quad (\text{A9})$$

where CPL is the limiting filter's spatial frequency in cycles/letter and MAR is the size of the letter stroke in min arc. If we assume that the resolution limit obtained using a point object is approximately the same as that obtained using letters, then equations A8 and A9 can be set equal to each other. Consequently, the relationship between MAR and  $\phi_{\text{min}}$  is:

$$\text{MAR} = \left( \frac{\pi \text{CPL}}{5 \times 3.83} \right) \phi_{\text{min}} \quad (\text{A10})$$

If we further assume that detection of a spatial frequency corresponding to 2.5 cycles/letter limits letter acuity, then the slope of the relationship (the factor in parentheses in equation A10) between MAR and  $\phi_{\text{min}}$  is 0.41.

## ACKNOWLEDGMENTS

HA was supported, in part, by Research Grant R01 EY08301 (National Eye Institute, National Institutes of Health, Bethesda, MD) to Kenneth Alexander while preparing the experiments; by Research Grant R01 EY01728 to Dennis Levi while conducting the experiments; and by NIH/National Institute on Aging Grant T32 AG00214, Interdisciplinary Research Training Program on Aging, University of Iowa, while preparing the manuscript. SSP was supported by a postdoctoral fellowship from the University of Houston Institute for

Space Systems Operation. Additional support for this study came from Research Grant R01 EY05068 to HEB; Core Center Grant P30 EY07551; and a faculty research grant from UHCO. We thank Dennis Levi for allowing us to use his laboratory to conduct these studies and Arthur Bradley for helpful insights into the relationship between resolution and the size of the retinal blur circle.

Received January 19, 2000; revision received July 14, 2000.

## REFERENCES

- Hopkins HH. The frequency response of a defocused optical system. Proc R Soc A 1955;231:91–103.
- Hopkins HH. Geometrical optical treatment of frequency response. Proc Phys Soc (Lond) 1957;70:1162–72.
- Fry GA. The optical performance of the human eye. In: Wolf E, ed. Progress in Optics, Vol 8. Amsterdam: North Holland, 1970: 53–131.
- Charman WN, Jennings JA. The optical quality of the monochromatic retinal image as a function of focus. Br J Physiol Opt 1976;31: 119–34.
- Smith G. Ocular defocus, spurious resolution and contrast reversal. Ophthalmic Physiol Opt 1982;2:5–23.
- Bradley A, Hong X, Chung ST, Thibos LN. The impact of defocus-induced phase reversals on letter recognition is different for hyperopes and myopes. Invest Ophthalmol Vis Sci 1999;40:S35.
- Thorn F, Schwartz F. Effects of dioptric blur on Snellen and grating acuity. Optom Vis Sci 1990;67:3–7.
- Herse PR, Bedell HE. Contrast sensitivity for letter and grating targets under various stimulus conditions. Optom Vis Sci 1989;66: 774–81.
- Bradley A, Hook J, Haeseker J. A comparison of clinical acuity and contrast sensitivity charts: effect of uncorrected myopia. Ophthalmic Physiol Opt 1991;11:218–26.
- Apkarian P, Tijssen R, Spekreijse H, Regan D. Origin of notches in CSF: optical or neural? Invest Ophthalmol Vis Sci 1987;28:607–12.
- Legge GE, Mullen KT, Woo GC, Campbell FW. Tolerance to visual defocus. J Opt Soc Am (A) 1987;4:851–63.
- Smith G, Jacobs RJ, Chan CD. Effect of defocus on visual acuity as measured by source and observer methods. Optom Vis Sci 1989;66: 430–5.
- National Research Council, National Academy of Sciences. Recommended standard procedures for the clinical measurement and specification of visual acuity. Report of working group 39. Committee on vision. Assembly of Behavioral and Social Sciences, National Research Council, National Academy of Sciences, Washington, DC. Adv Ophthalmol 1980;41:103–48.
- Pelli DG, Zhang L. Accurate control of contrast on microcomputer displays. Vision Res 1991;31:1337–50.
- Finney DJ. Probit Analysis. 3rd ed. Cambridge: Cambridge University Press, 1971.
- Watson AB, Pelli DG. QUEST: a Bayesian adaptive psychometric method. Percept Psychophys 1983;33:113–20.
- Woods RL, Bradley A, Atchison DA. Monocular diplopia caused by ocular aberrations and hyperoptic defocus. Vision Res 1996;36: 3597–606.
- Ginsburg AP. Visual Information Processing Based on Spatial Filters Constrained by Biological Data (AMRL-TR; 78-129). Wright-Patterson Air Force Base, OH: Aerospace Medical Research Laboratory, Air Force Systems Command, 1978.
- Legge GE, Pelli DG, Rubin GS, Schleske MM. Psychophysics of reading—I. Normal vision. Vision Res 1985;25:239–52.
- Parish DH, Sperling G. Object spatial frequencies, retinal spatial

- frequencies, noise, and the efficiency of letter discrimination. *Vision Res* 1991;31:1399–415.
21. Alexander KR, Xie W, Derlacki DJ. Spatial-frequency characteristics of letter identification. *J Opt Soc Am (A)* 1994;11:2375–82.
  22. Bondarko VM, Danilova MV. What spatial frequency do we use to detect the orientation of a Landolt C? *Vision Res* 1997;37:2153–6.
  23. Solomon JA, Pelli DG. The visual filter mediating letter identification. *Nature* 1994;369:395–7.
  24. Anderson RS, Thibos LN. Sampling limits and critical bandwidth for letter discrimination in peripheral vision. *J Opt Soc Am (A)* 1999;16:2334–42.

**Harold E. Bedell**

*College of Optometry*

*University of Houston*

*Houston, TX 77204-6052*

*e-mail: HBedell@mail-gw.opt.uh.edu*